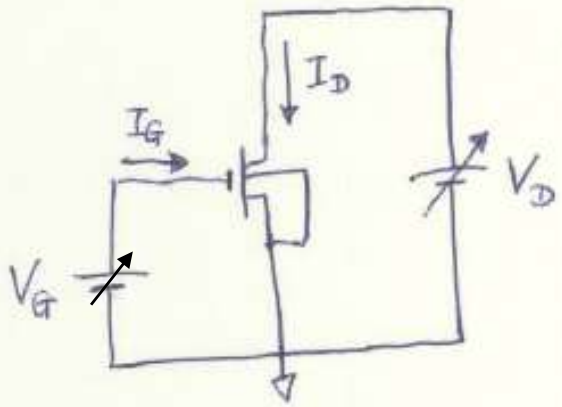
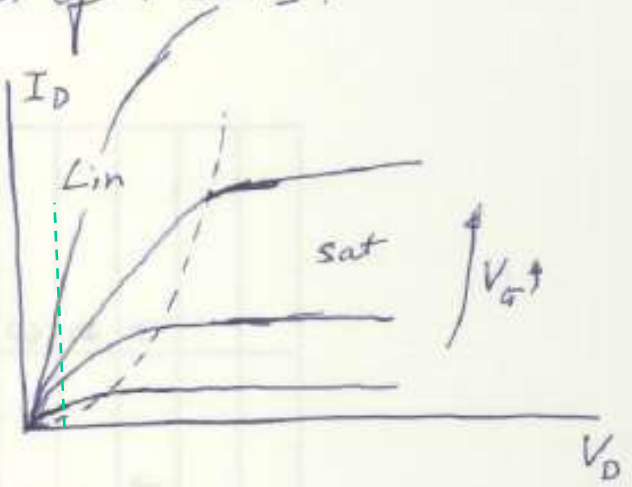
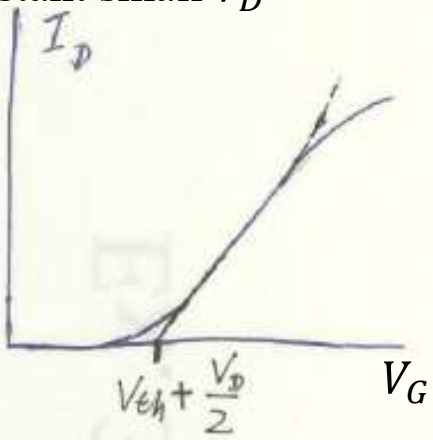
□ I-V Characterization of FETs





@ constant small V_D



Lin:

Oxide capacitance per area

$$I_D = \mu C_{ox} \frac{W}{L} V_D \left[(V_G - V_{th}) - \frac{V_D}{2} \right]$$

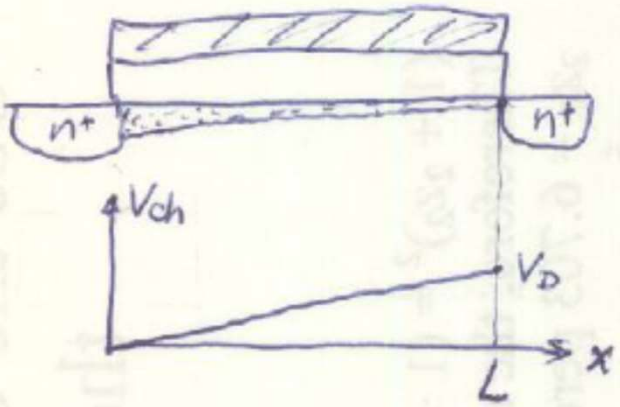
W/L ratio, or reciprocal of # of □s

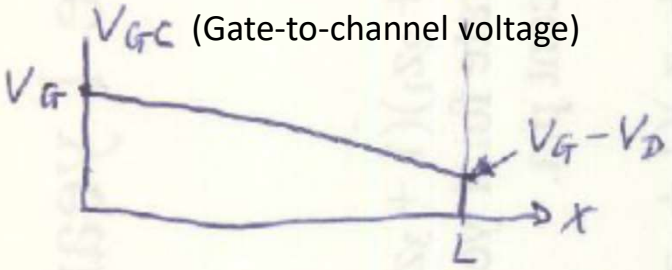
But this eq. is about ideal FETs.

Should choose $V_D \ll V_{th}$.

So, $V_{th} + \frac{V_D}{2} \approx V_{th}$

Transconductance $g_m = \frac{dI_D}{dV_G} = \mu C_{ox} \frac{W}{L} V_D$





At larger V_D:

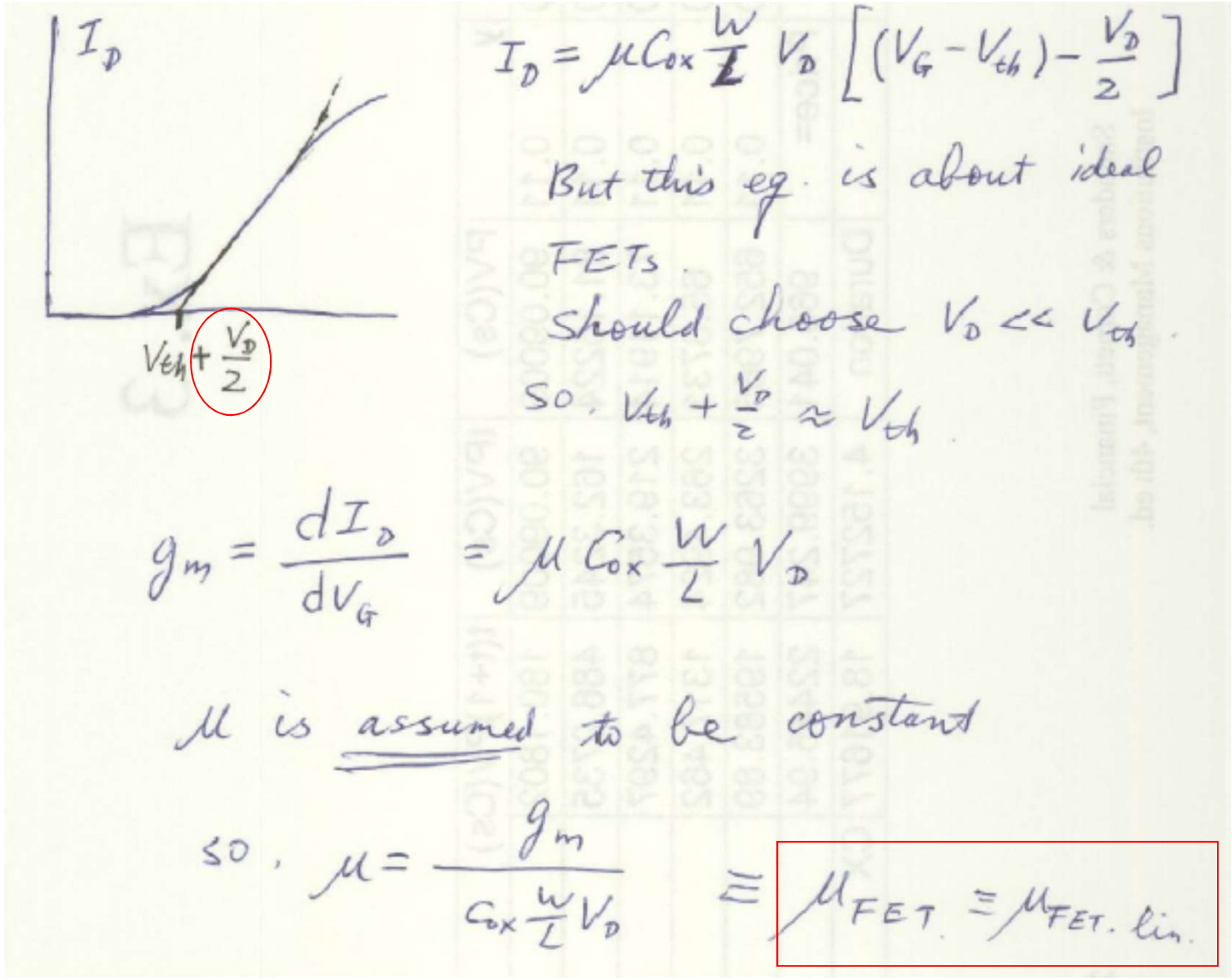
$n(x)$ ↓ as $x \to L$.

So, output curves bend down.

$$V_{\text{ch}}(x) = \int_0^x I_D \left(\frac{L}{W}\right) \frac{1}{n(x')q\mu} dx' = \frac{1}{q\mu}\left(\frac{W}{L}\right) I_D \int_0^x \frac{1}{C_{\text{ox}}(V_{\text{ch}}(x') - V_{th})} dx'$$

Solve this equation → $V_{\text{ch}}(x)$

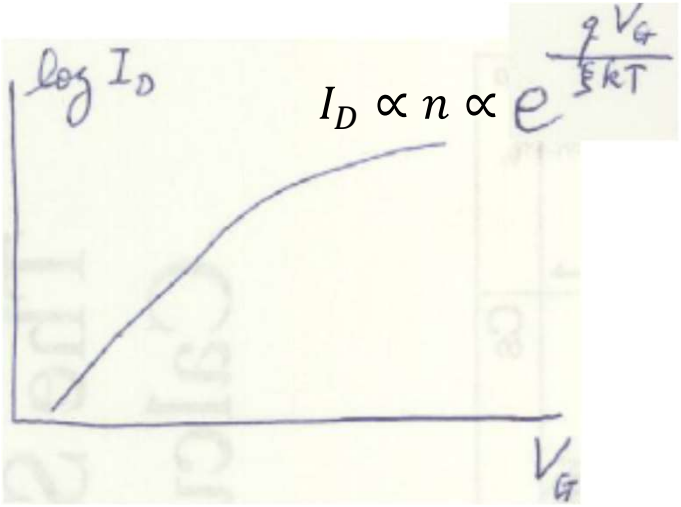
$$V_{\text{D}} = V_{\text{ch}}(L)$$



This extracted mobility is just a scaled version of g_m, a device performance parameter, **not** a semiconductor material property.

Tables in device textbooks list an electron mobility of ~1000 cm^2/Vs for Si, but values in models for circuit simulation are always much lower.

Subthreshold



$$\xi = 1 + \frac{C_d}{C_{ox}}$$

Subthreshold slope

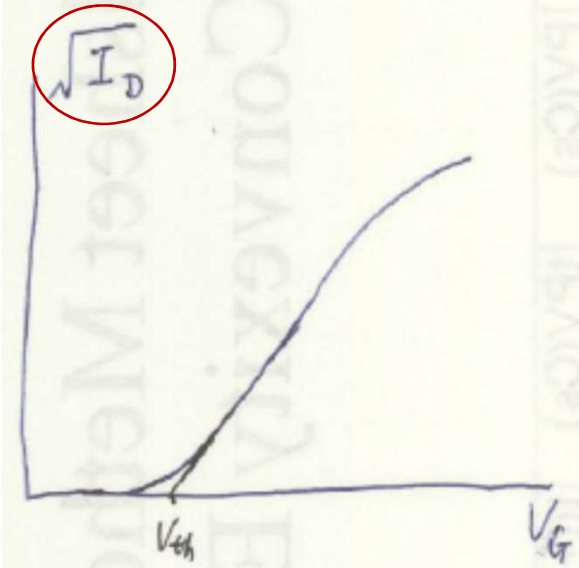
$$\frac{d}{dV_G} \log I_D \leq \frac{1}{60} \text{ dec/mV}$$

Subthreshold swing

$$\frac{dV_G}{d(\log I_D)} \leq 60 \text{ mV/dec}$$

Saturation

(choose V_D to ensure $V_D > V_G - V_{th}$)



$$I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_G - V_{th})^2$$

$$\text{slope} = \sqrt{\frac{1}{2}\mu C_{ox}\frac{W}{L}}$$

Extracted parameters: $V_{th,\text{sat}}$, μ_{sat}

For a real FET, do **not** expect

$$\mu_{lin} = \mu_{sat}, \quad \text{or} \quad V_{th,lin} = V_{th,sat}$$

These are just extracted parameters.

Homework 5(?)

Virtual characterization of MOSFETs

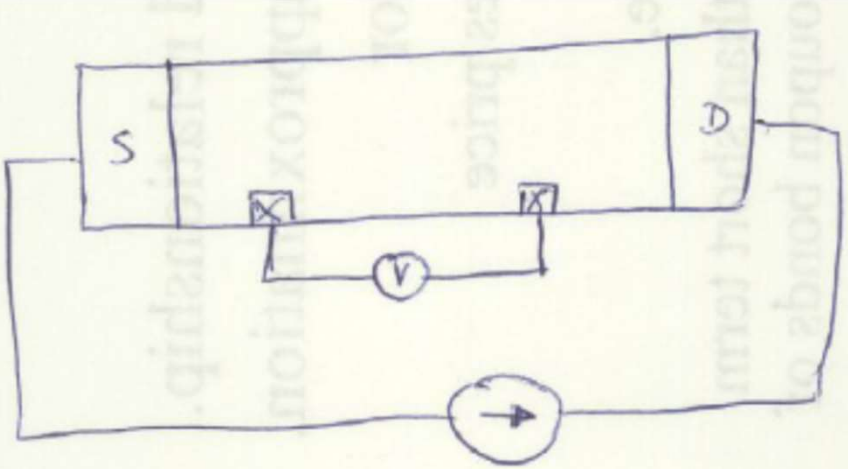
If you are more concerned about the physical properties of the channel semiconductor, characterize the FET in the linear regime at very low V_D.

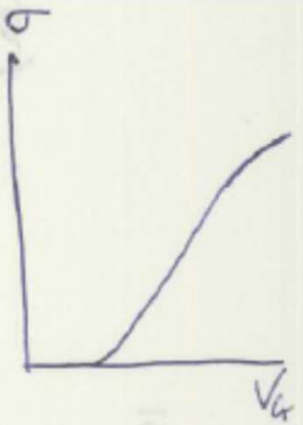
Basically measure channel resistance vs. V_G.

If μ is constant, $I_D \propto \mu C_{ox}(V_{GS} - V_{th})$

For certain materials, contact resistance may be significant.

To minimize the error caused by contact resistance, use 4-probe measurement.





$$\sigma = \frac{I_D}{V_D} \cdot \frac{L}{W} = \mu C_{ox}(V_G - V_{th}) = q n \mu$$

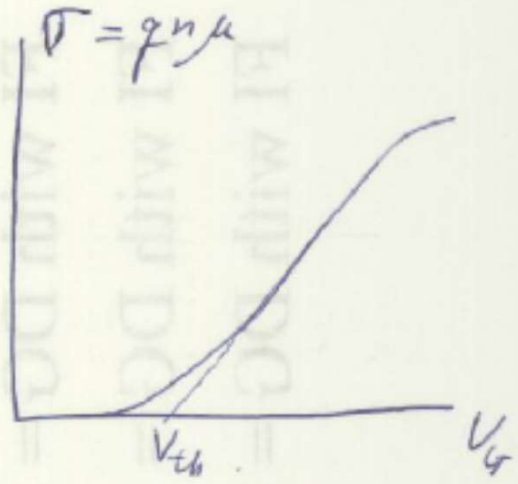
2D σ: Ω; 2D n: cm^{-2}

Even in careful 4-probe measurement, we are actually measure the product

μ_n vs. V_G.

V_{th} is unknown and must be extracted.

So. we can't accurately obtain n.



We assume μ is constant, and obtain it's value from the slope.

$$\mu_{FET} = \frac{1}{q}\frac{d\sigma}{dn}$$

$$\sigma = \mu C_{ox}(V_G - V_{th})$$

$$\frac{d\sigma}{dV_G} = \mu C_{ox} \Rightarrow \mu_{FET} = \frac{1}{C_{ox}}\frac{d\sigma}{dV_G}$$

$$\sigma = \mu C_{ox} (V_G - V_{th})$$

$$\frac{d\sigma}{dV_G} = \mu C_{ox} \Rightarrow \mu_{FET} = \frac{1}{C_{ox}} \frac{d\sigma}{dV_G}$$

What if $\frac{d\mu}{dn} \neq 0$, i.e. $\frac{d\mu}{dV_G} \neq 0$?

$$\frac{d\sigma}{dV_G} = \mu C_{ox} + C_{ox} (V_G - V_{th}) \frac{d\mu}{dV_G}$$

You cannot get $\mu = \frac{1}{C_{ox}} \frac{d\sigma}{dV_G}$!!!

If you know $\mu = \mu(n)$ strongly depends on n, this is a better way:

Extract V_{th} first
(a little bit arbitrary)

$$q_n = C_{ox}(V_G - V_{th})$$

Then, $$\mu = \frac{\sigma}{q_n}$$

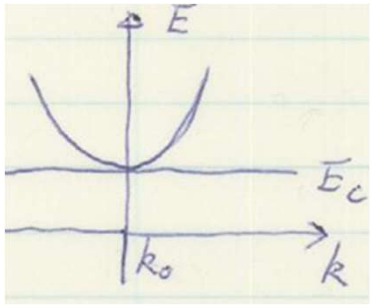
To truly disentangle μ and n, we need to perform Hall effect measurement, which yields n and σ. Then, we have

$$\mu = \frac{\sigma}{q_n}$$

We have discussed the big picture of Si MOSFETs.
To better understand their operation and the caveats in FET characterization, we need a bit more semiconductor physics.

The semi-classical model re-visited

1D heuristic



Almost always a good approximation near a band minimum (Taylor expansion):

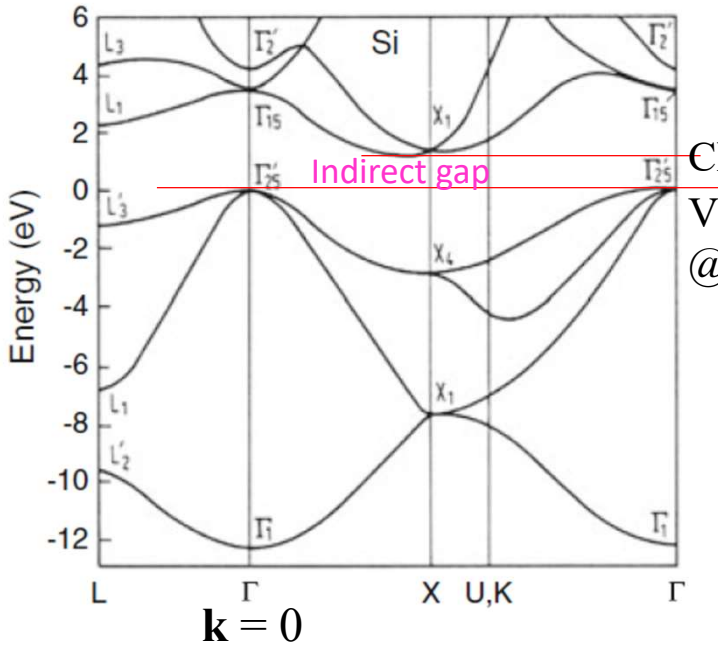
$$E(k) = E(k_0) + \frac{1}{2}\frac{\partial^2 E}{\partial k^2}\bigg|_{k=k_0} (k-k_0)^2$$

If we consider $E(k_0) = E_C$ as a potential energy V, and $\hbar(k-k_0)$ as a momentum p, then this becomes formally the same as a classical particle:

$$E = \frac{p^2}{2m} + V \quad \Leftrightarrow \quad E = \frac{\hbar^2(k-k_0)^2}{2m_e^*} + E_c$$

The "effective mass" m_e^* is given by:

$$\frac{1}{m_e^*} = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k^2}\bigg|_{k=k_0}$$



Si band structure

Anisotropic curvature (effective mass)

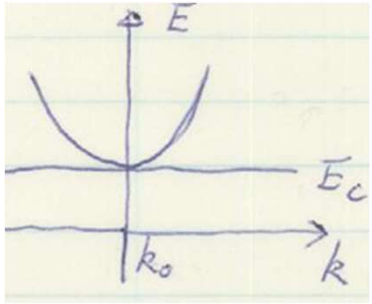
$$[(m_e^*)^{-1}]_{ij} = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k_i \partial k_j}\bigg|_{k=k_0}$$

Take average

$$(m_e^*)^{-1} = \frac{[(m_e^*)^{-1}]_{xx} + [(m_e^*)^{-1}]_{yy} + [(m_e^*)^{-1}]_{zz}}{3}$$

(good approx. near a minimum)

1D heuristic



$$E(k) = E(k_0) + \frac{1}{2}\frac{\partial^2 E}{\partial k^2}\bigg|_{k=k_0} (k-k_0)^2$$

$$E(k_0) = E_C$$

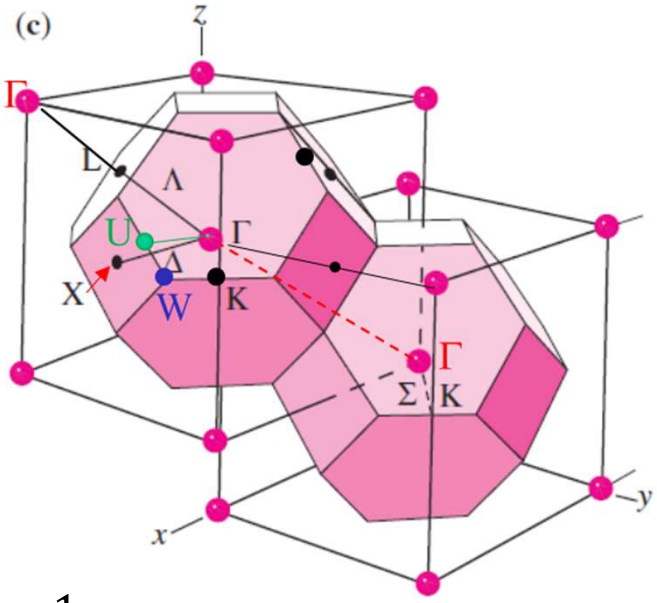
$$E = \frac{\hbar^2(k-k_0)^2}{2m_e^*} + E_C$$

Electron moves at group velocity of Bloch wave packet

$$v = \frac{1}{\hbar}\frac{dE}{dk} = \frac{1}{\hbar}\frac{dE}{d(k-k_0)} = \frac{\hbar(k-k_0)}{m_e^*} = \frac{p}{m_e^*}$$

$$E = E_C + \frac{1}{2}m_e^* v^2$$

Si



$\mathbf{k}_0$ along ΓX, near X

$$E(\mathbf{k}) = E(\mathbf{k}_0) + \frac{1}{2}(m_e^*)^{-1}\hbar^2(\mathbf{k}-\mathbf{k}_0)^2$$

$$E(\mathbf{k}_0) = E_C$$

$$\mathbf{v} = \frac{1}{\hbar}\nabla_{\mathbf{k}}E = \frac{1}{\hbar}\left(\hat{\mathbf{x}}\frac{\partial E}{\partial k_x} + \hat{\mathbf{y}}\frac{\partial E}{\partial k_y} + \hat{\mathbf{z}}\frac{\partial E}{\partial k_z}\right)$$

$$= \hat{\mathbf{x}}v_x + \hat{\mathbf{y}}v_y + \hat{\mathbf{z}}v_z$$

$$= \frac{\hbar(\mathbf{k}-\mathbf{k}_0)}{m_e^*} = \frac{\mathbf{p}}{m_e^*}$$

$$E = E_C + \frac{1}{2}m_e^* v^2 = E_C + \frac{1}{2}m_e^*\left(v_x^2 + v_y^2 + v_z^2\right)$$

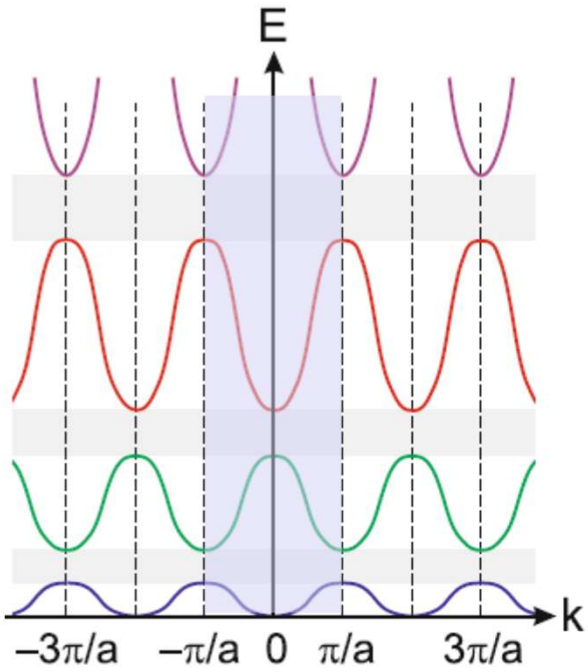
1D heuristic

"Newton's 2nd law"

$$\hbar\frac{dk}{dt} = \hbar\frac{d}{dt}(k - k_0) = F = -q\mathcal{E}$$

Force on electron

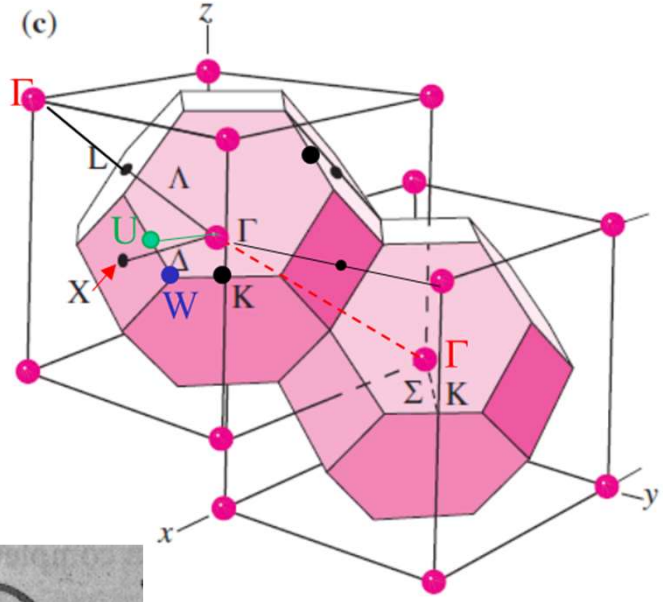
Were there no scattering, Bloch oscillation

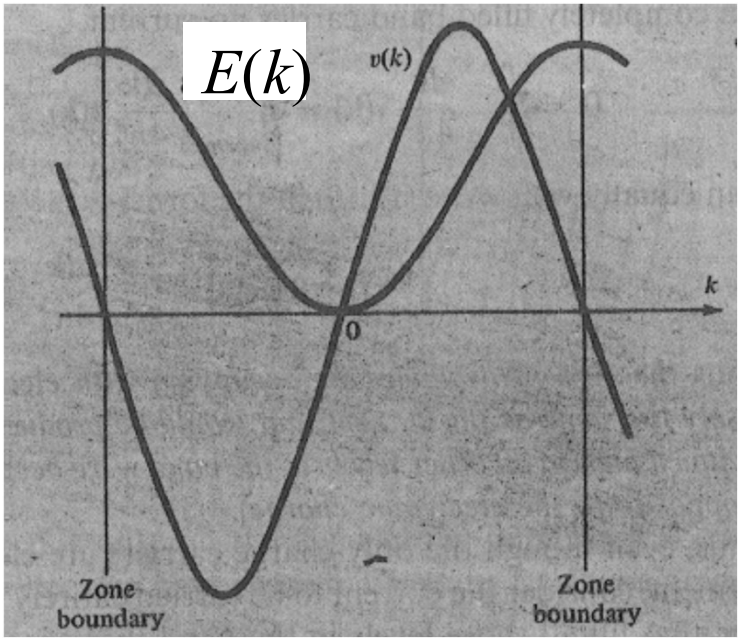


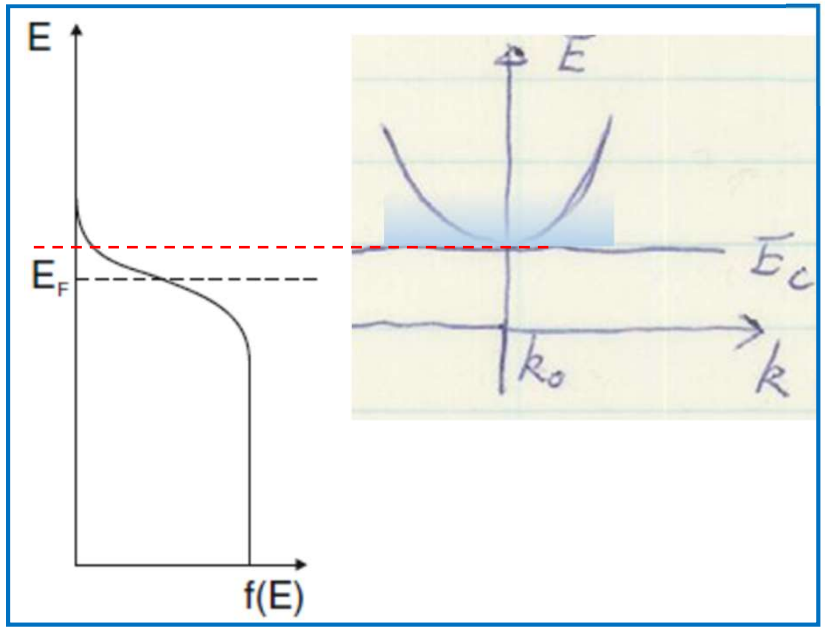
Si

$$\hbar\frac{d\mathbf{k}}{dt} = \hbar\frac{d}{dt}(\mathbf{k} - \mathbf{k}_0) = \mathbf{F} = -q\mathcal{E}$$

Si BZ







In a semiconductor,

- electrons constantly undergo scattering,
- at average time interval $\tau \sim 10^{-14}$ s (relaxation time);
- frequent collisions help maintain thermal equilibrium.
- At thermal equilibrium, with $E_C - E_F > 4k_BT$, electrons follow Boltzmann distribution, as an ideal gas.
- In a non-degenerate semiconductor, all electrons are near conduction band bottom $\Rightarrow$ constant m_e^*.

Then root-mean-square velocity, or thermal velocity v_{th}:

$$\frac{1}{2}m_e^* v_{th}^2 = \frac{1}{2}k_BT \qquad \text{1D}$$

$$\frac{1}{2}m_e^* v_{th}^2 = \frac{3}{2}k_BT \qquad \text{3D}$$

For Si, $v_{th} \sim 2.3 \times 10^7$ cm/s. Compare: $c = 3 \times 10^8$ m/s $= 3 \times 10^{10}$ cm/s

When an external electric field is applied, each electron gains an average net momentum $\hbar\Delta k$.

$$m^* v_d = \hbar\Delta k = F\tau = -q\mathcal{E}\tau \Rightarrow \quad v_d = -\frac{q\mathcal{E}\tau}{m^*}$$

Define mobility $\quad \mu = -\frac{q\tau}{m^*}$

Then, drift velocity $v_d = \mu\mathcal{E}$

(You may use a different convention to make the electron mobility positive.)

$$J = -nqv_d = nq\mu\mathcal{E} = \sigma\mathcal{E} \qquad (1)$$

No matter what convention for the sign of electron mobility you use, Ohm's law remains the same.

For FETs, the channel is (quasi-)2D. We use J_{2D} (current per width, A/cm), n_{2D} (carriers per area, cm^{-2}), and σ_{2D} (2D conductivity, Ω^{-1} □; $1/\sigma_{2D} \equiv \rho_{2D}$ is 2D resistivity or sheet resistance, Ω/ □).

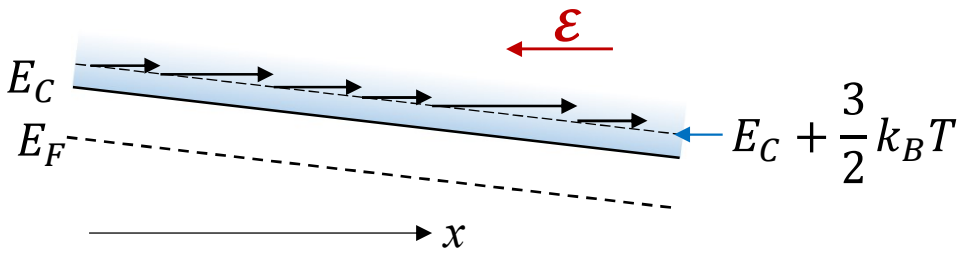
$$J_{2D} = \int_0^{-\infty} J(z)dz = q\mu\mathcal{E}\int_0^{-\infty} n(z)dz = n_{2D}q\mu\mathcal{E} = \sigma_{2D}\mathcal{E}$$

$$\sigma_{2D} = n_{2D}q\mu$$

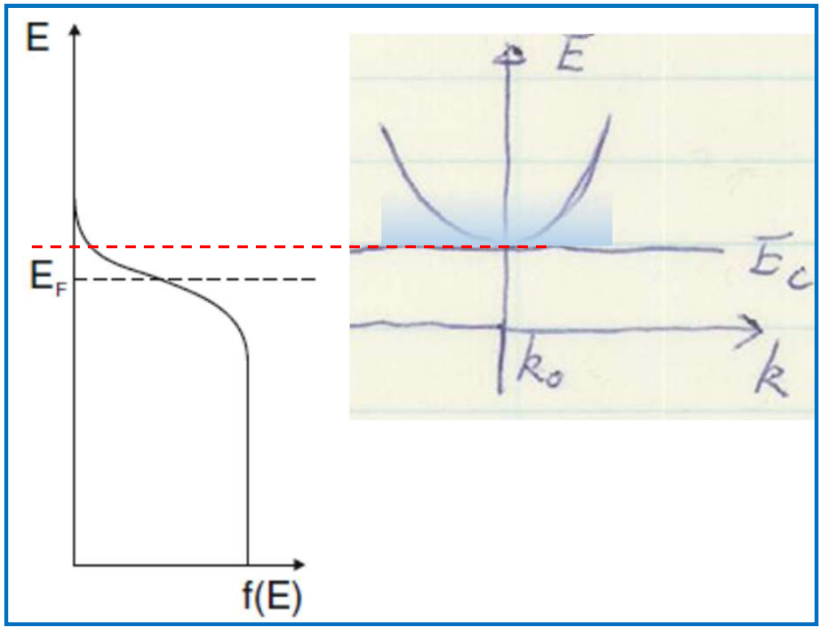
These equations look the same as Eq. (1). So, we just drop the subscripts "2D" and simply write Eq. (1). Just keep in mind that we are dealing w/ the 2D quantities.

We stopped here on Tue 9/28/2021.

At low electric fields, small disturbance to thermal equilibrium.



Schematic illustration of the relaxation time approximation



E_C is like the potential energy of e.
Slope of energy levels is $q\mathcal{E}$.
Electric field applied ➔ nonequilibrium.
Electrons and lattice locally in equilibrium ➔ local $E_F(x)$ can be defined.

Scattering (collision) mechanism include all deviations from stationary, perfect crystal structure: defects and lattice vibration.

Different scattering processes have different contributions to the total scattering rate (probability of scattering per time):

$$\frac{1}{\tau} = \sum_i \frac{1}{\tau_i}$$

The scattering rate or frequency due to the *i*th process/mechanism

The total scattering rate is a simple sum of rates of individual processes **if** the processes are independent of each other.

We can then assign $\mu_i = \frac{q\tau_i}{m^*}$ thus $\frac{1}{\mu} = \sum_i \frac{1}{\mu_i}$

For a given T, μ decreases with increasing doping due to impurity scattering.

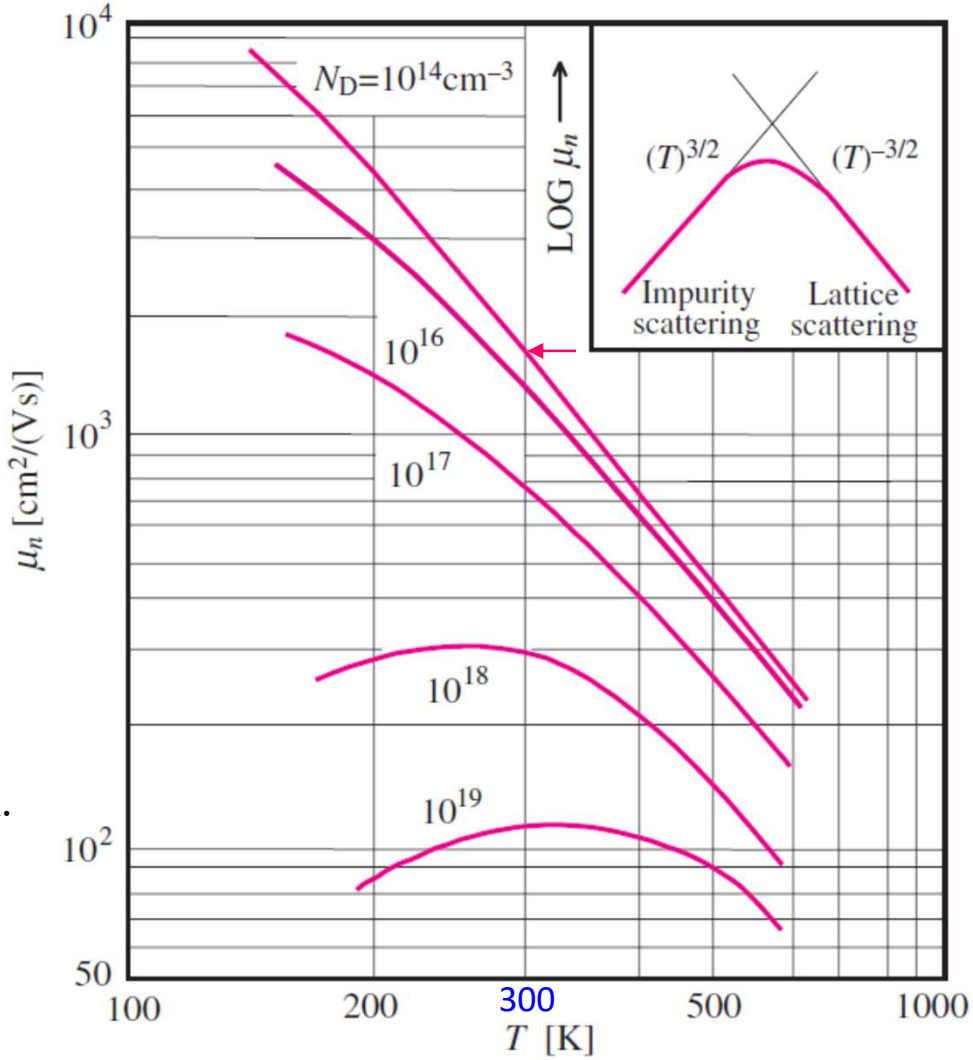
For a mature semiconductor e.g. Si, the major defect scattering mechanism is charged dopant impurity scattering.

Impurity scattering rate decreases with increasing T (higher thermal velocity), since faster e less likely to be scattered by charged impurities.

Lattice scattering rate increases with increasing T (stronger vibration).

A quantum of vibration is called a phonon. Thus lattice vibrations scattering is also called phonon scattering.

The textbook listed $\mu_e = 1500\ \text{cm}^2/(\text{Vs})$ is for low doping at 300 K (indicated by ←).

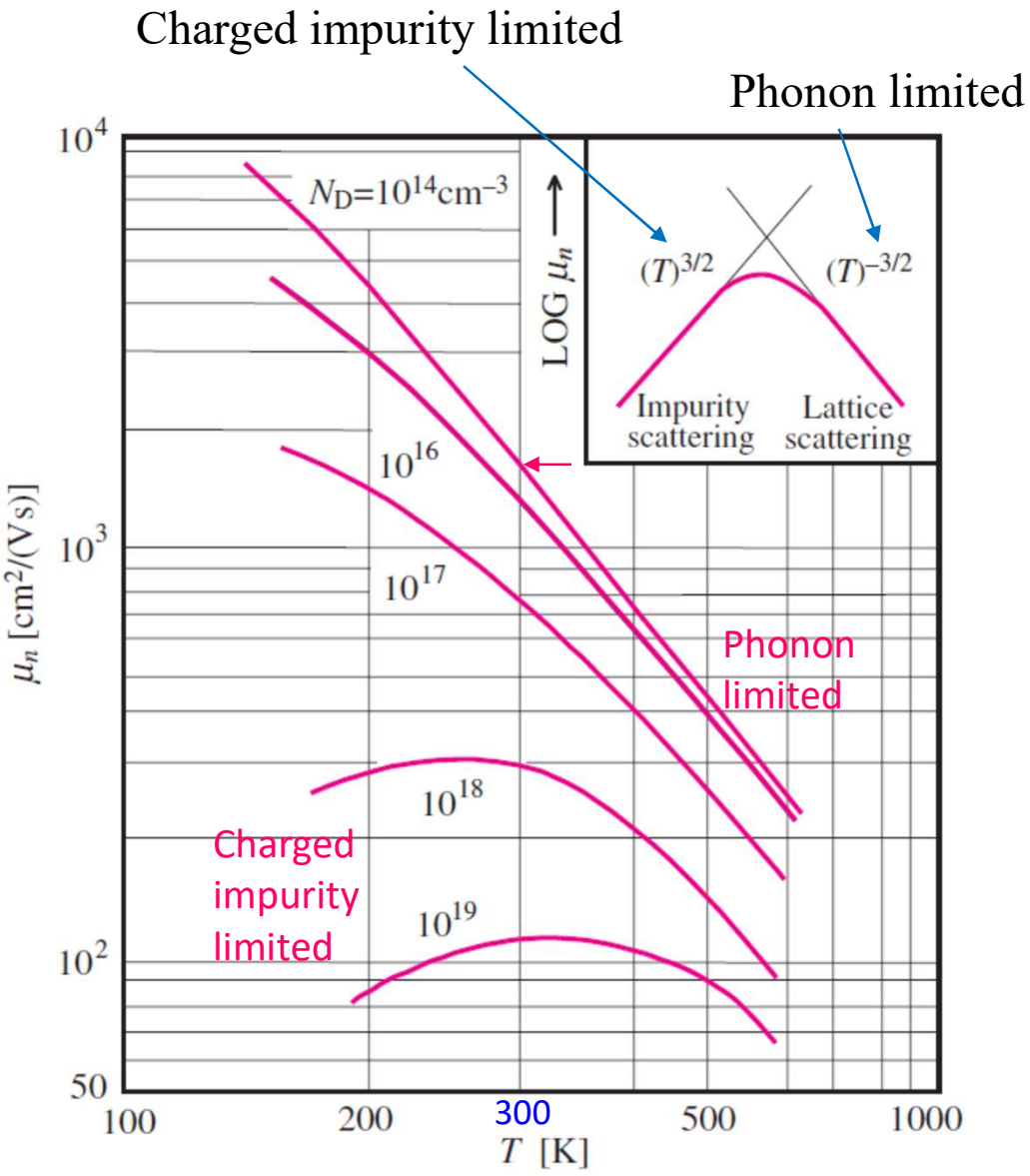


Temperature dependence of mobilities in n-type Si

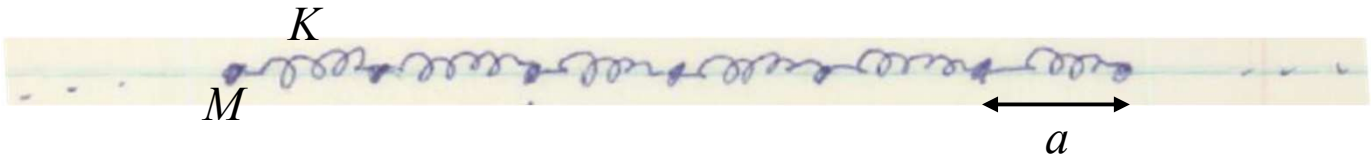
Yu & Cardona, *Fundamentals of Semiconductors*, 4th Ed. p. 222

High doping at low T, mobility is charged impurity limited.

Low doping at high T, mobility is phonon limited.



Phonons



Model of 1D mono-atomic crystal

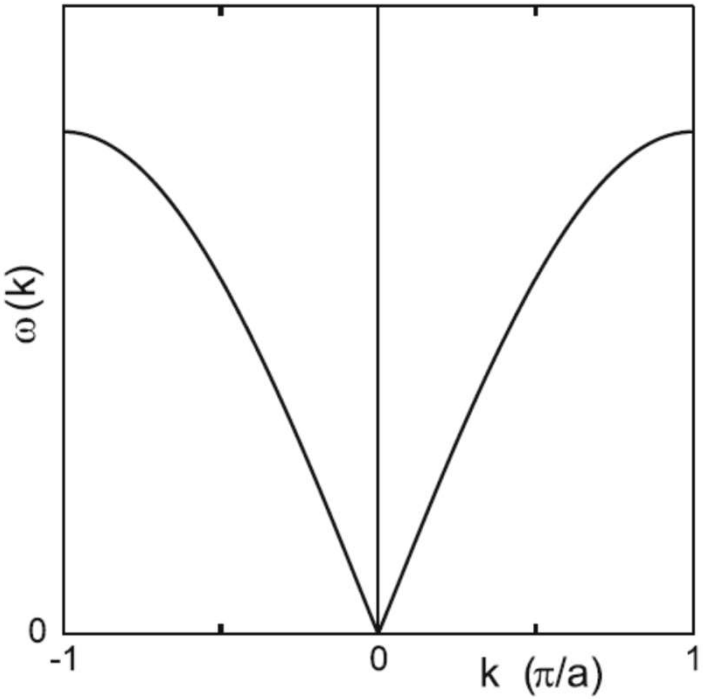
Vibration will propagate through the chain, just as in a continuous medium – sound waves:

Each (ω, k) is a vibration mode, whose energy quantum (called a phonon) is $\hbar\omega$.

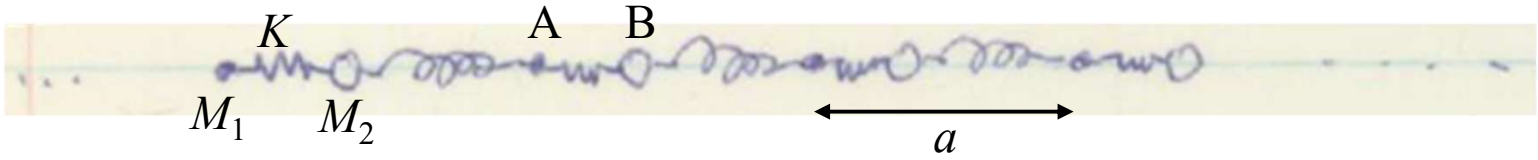
For long waves, small wavevectors k, $\lambda \gg a$, as if the chain is a continuous string. The speed of the sound wave $\omega/k \approx d\omega/dk$.

Discrete nature of lattice shows up for short wave (larger k, higher ω).

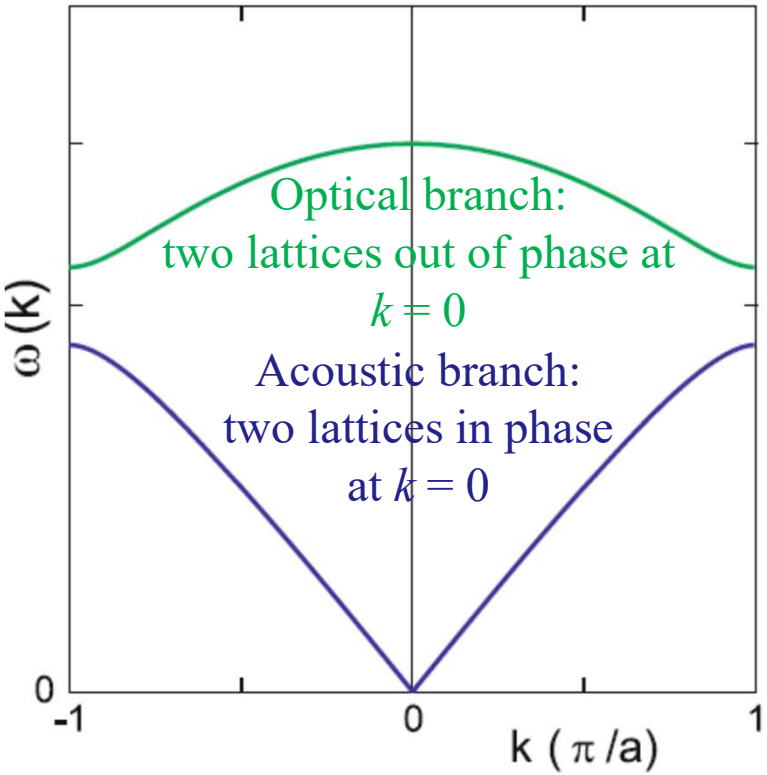
We stopped here on Thu 10/5/2021.



Periodicity ➔
Vibration modes of k and $k + G = k + n(2\pi/a)$ are indistinguishable. ➔
Phonon dispersion is presented only in the 1st BZ, as for electronic band structures.



1D diatomic crystal



Each (ω, k) is a vibration mode, whose energy quantum (called a phonon) is $\hbar\omega$.

High phonon energy

Low phonon energy

There are no continuous solid analogy of optical phonons.

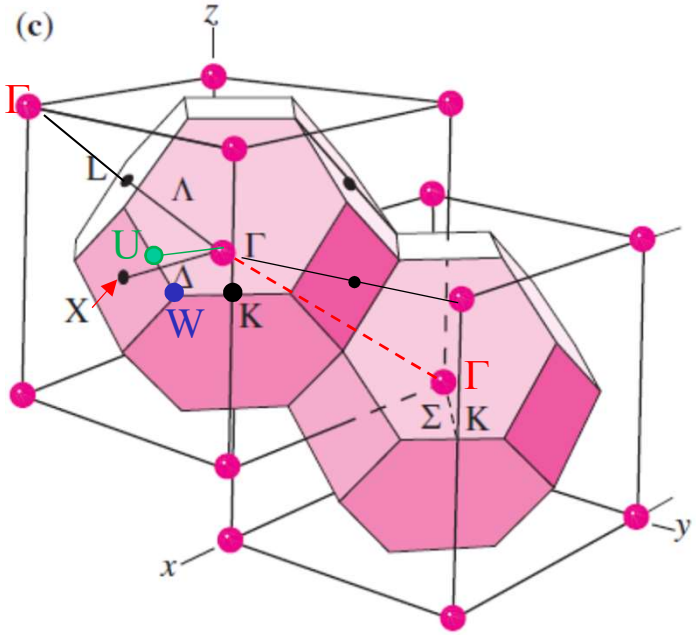
Watch optical animation: https://www.youtube.com/watch?v=M4WQs_U1nmU

Now, let's look at real crystals in 3D. Example: Si.

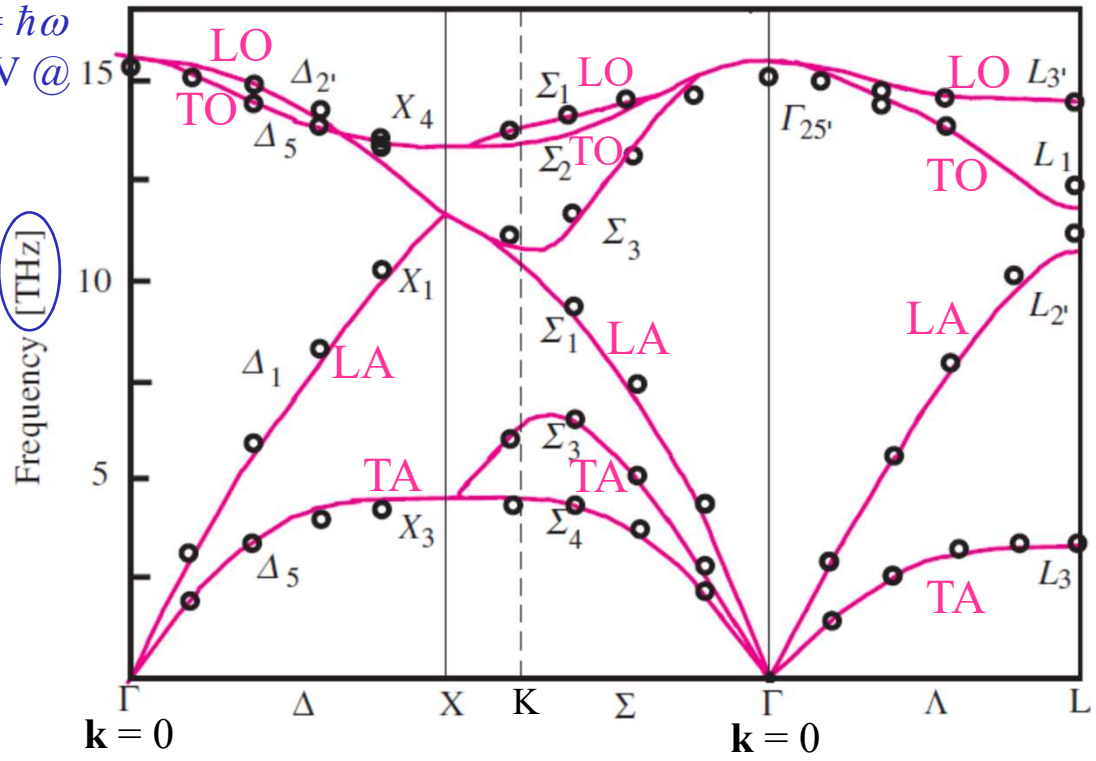
Recall that the Si primitive unit cell contains two atoms (inequivalent although both are Si).

LA: longitudinal acoustic
TA: transverse acoustic
LO: longitudinal optical,
LO: transverse optical.

$E = h\nu = \hbar\omega$
= 62 meV @



Curves: calculated. Circles: measured.



Yu & Cardona, *Fundamentals of Semiconductors*, 4th Ed. p. 111

The dispersion is plotted only along high symmetry directions.
Again, notice linear dispersion near **k** = 0.
What is the physical meaning of the slopes?

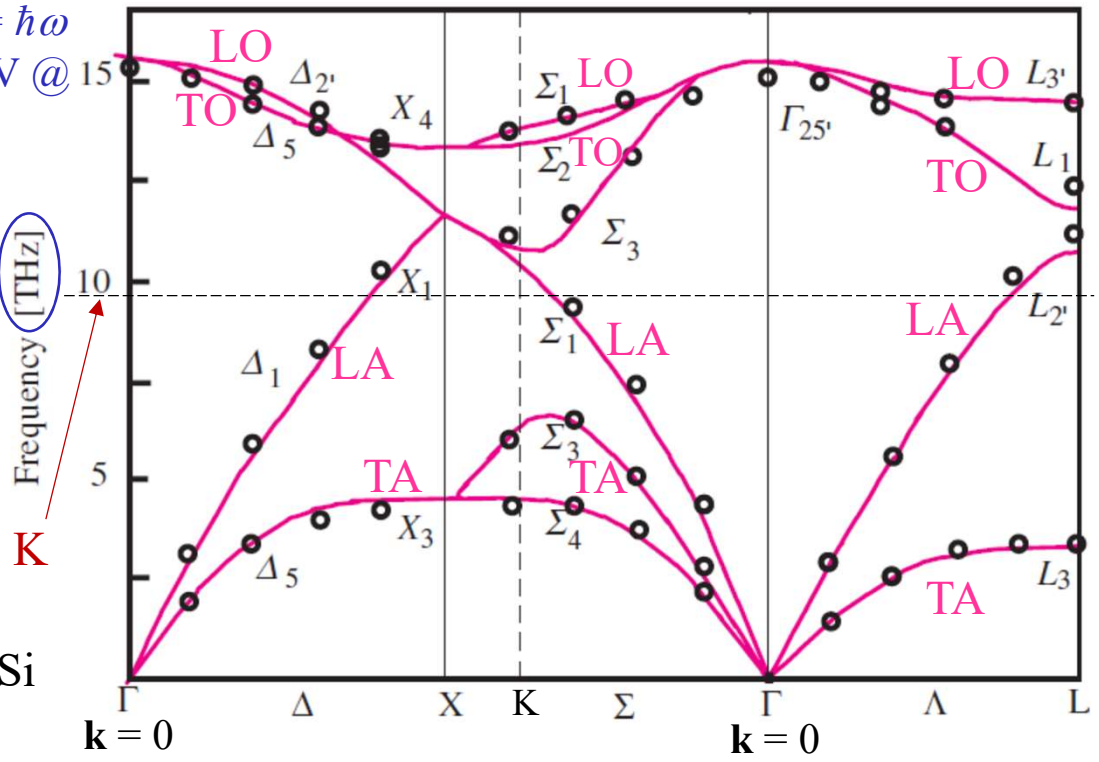
Photon and phonon energy can be expressed in many ways:

Energy	Frequency	Wavelength (photon)	Wave number (photon)
1 eV	241.8 THz	1240 nm	8066 cm^{-1}
62 meV	15 THz	20 μm	500 cm^{-1}

See http://halas.rice.edu/conversions

$E = h\nu = \hbar\omega$
= 62 meV @ 15

Curves: calculated. Circles: measured.



An electron cannot emit (or absorb) phonons with higher energy than its own.

$$\frac{1}{2}m_e^* v_{th}^2 = \frac{3}{2}k_B T$$

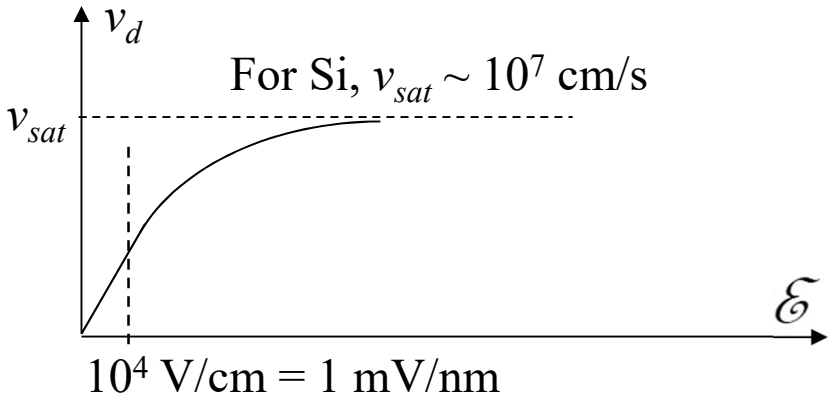
38 meV (~9 THz) at 300 K

Under low electric fields, electrons in Si interact only with acoustic phonons.

Under **high fields**, carriers gain energy faster than they lose to scattering. ⇒
Electrons & the lattice not in equilibrium.
Electrons are hotter than the lattice. ⇒ Hot electrons.

When the energy of hot electrons becomes comparable to that of optical phonons (62 meV for Si), energy is transferred to the lattice via optical phonons.

New, high-rate scattering mechanism kicks in ⇒ velocity saturation



This is much lower than in a MOSFET channel in saturation regime.
The drain current cannot increase indefinitely with increasing V_D/L.

Implication of velocity saturation: Saturation regime

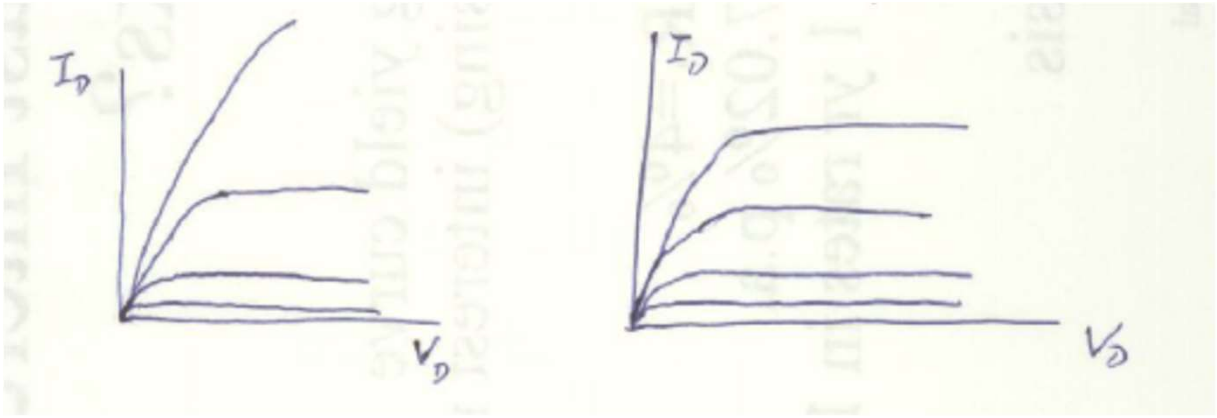
$$Q = C_{ox} W L (V_G - V_{th})$$ Total charge in channel, not per area

With channel pinched off, there is a factor such as 2/3, but this does not affect the big picture, as discussed later.

$$I_D = Q \cdot \frac{v_{sat}}{L} = C_{ox} W (V_G - V_{th}) v_{sat}$$

Here, $I_D \propto V_G - V_{th}$

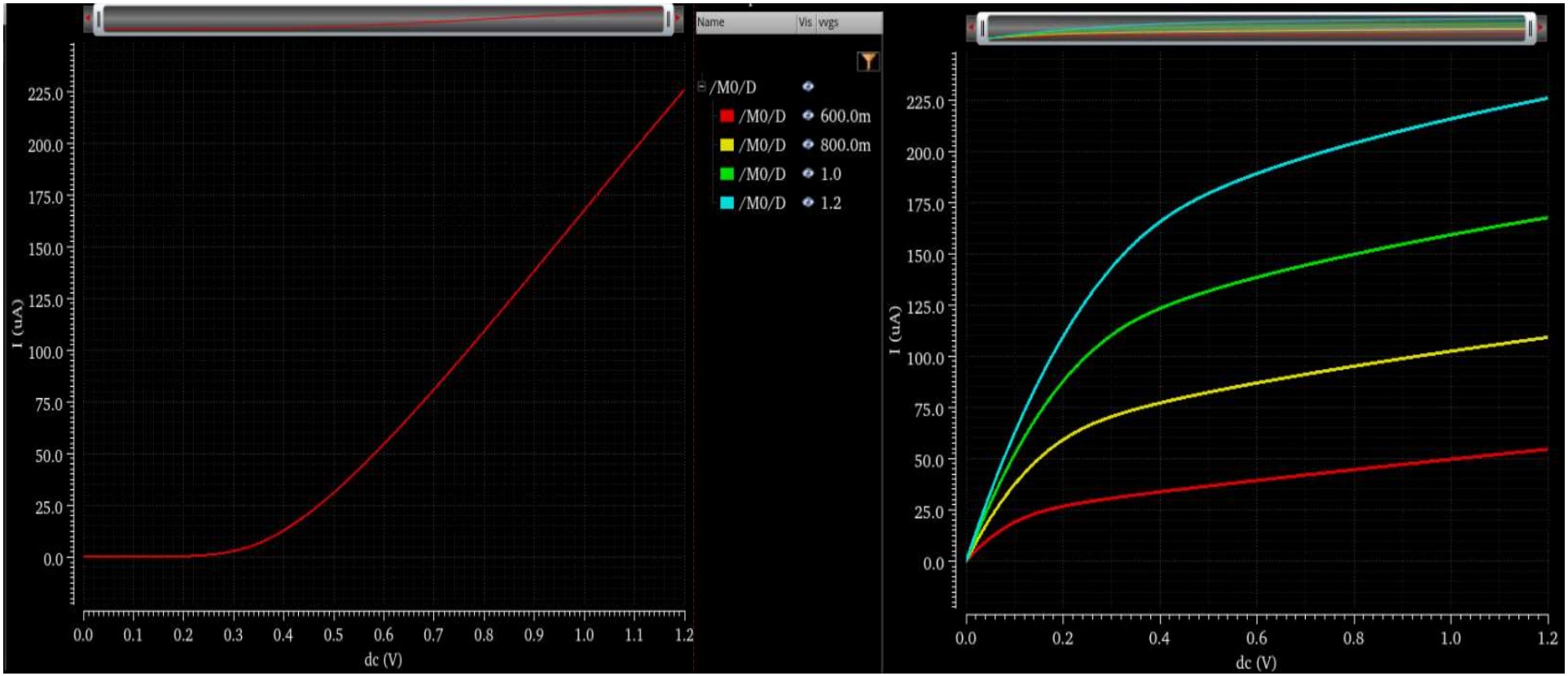
Compare this to $I_D \propto (V_G - V_{th})^2$ where saturation is dew to pinch-off only.



Considering pinch-off only

With velocity saturation

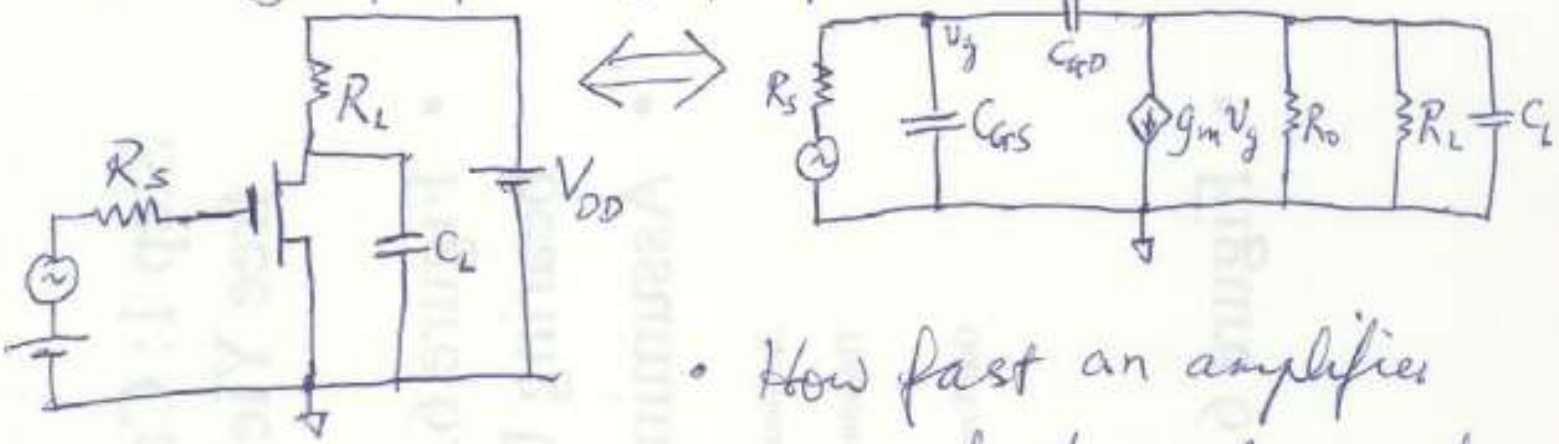
Velocity saturation impacts high-speed, high-power operations.



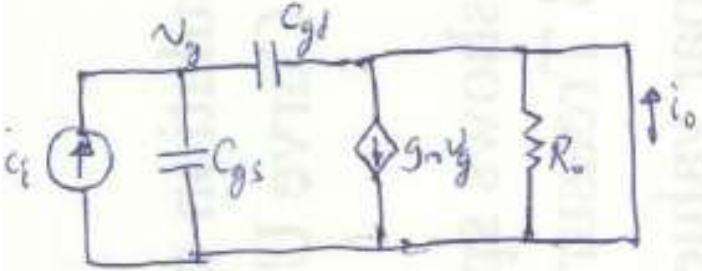
Transfer characteristics: I_D – V_{gs} curve at V_{ds}=1.2 V

Output characteristics: I_D- V_{ds} for different values of Vgs (0.6, 0.8, 1 and 1.2 V)

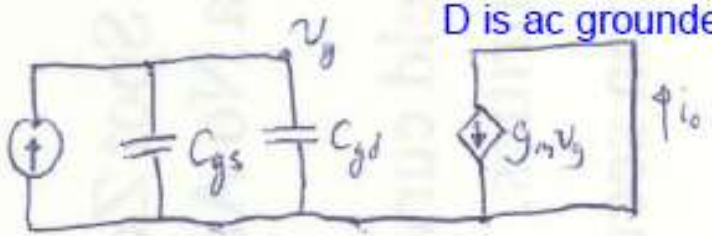
☐ High frequency performance



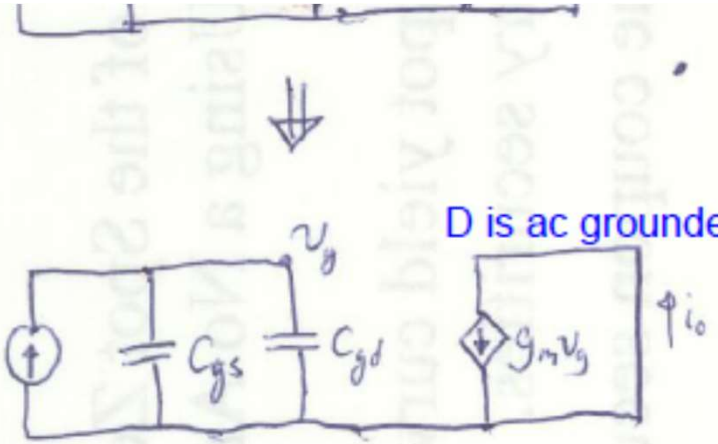
- How fast an amplifier can work depends on the ckt: R_s, R_L, C_L etc.
- Need a "Figure of Merit" for the FET device for fair comparison
- The Figure of Merit f_T: the frequency at which the current gain is 0dB (i.e. unity) when driving no load.



D is ac grounded



$$g_m = \frac{dI_D}{dV_G} = \mu C_{ox} \frac{W}{L} (V_G - V_{th})$$



- The Figure of Merit f_T: the frequency at which the current gain is 0dB (i.e. unity) when driving no load.

$$g_m = \frac{dI_D}{dV_G} = \mu C_{ox} \frac{W}{L} (V_G - V_{th})$$

$$v_g = i_i \frac{1}{j\omega C_{gs}} \qquad (C_{gd} \text{ due to G-D overlap, ignored})$$

$$i_o = g_m v_g = i_i \frac{g_m}{j\omega C_g}, \quad \left|\frac{i_o}{i_i}\right| \propto \frac{1}{\omega}$$

when $\left|\frac{i_o}{i_i}\right| = 1$

$$\omega_T = \frac{g_m}{C_g} = \frac{\mu C_{ox} \frac{W}{L} (V_G - V_{th})}{\frac{2}{3} C_{ox} WL} = \frac{\mu (V_G - V_{th})}{\frac{2}{3} L^2}$$

$$f_T = \frac{\omega_T}{2\pi}$$

$$\omega_T = \frac{\mu (V_G - V_{th})}{\frac{2}{3} L^2}$$

$\mu \uparrow$, $L \downarrow$ $\Rightarrow \omega_T \uparrow$

Assumed saturation. $V_D > V_G - V_{th}$.

The higher $V_G - V_{th}$, the harder you drive the carriers by higher V_D (to stay in saturation)

$(v_{drift} \propto \mu \frac{V_D}{L})$

However, the drift velocity cannot increase indefinitely with increasing V_D/L.

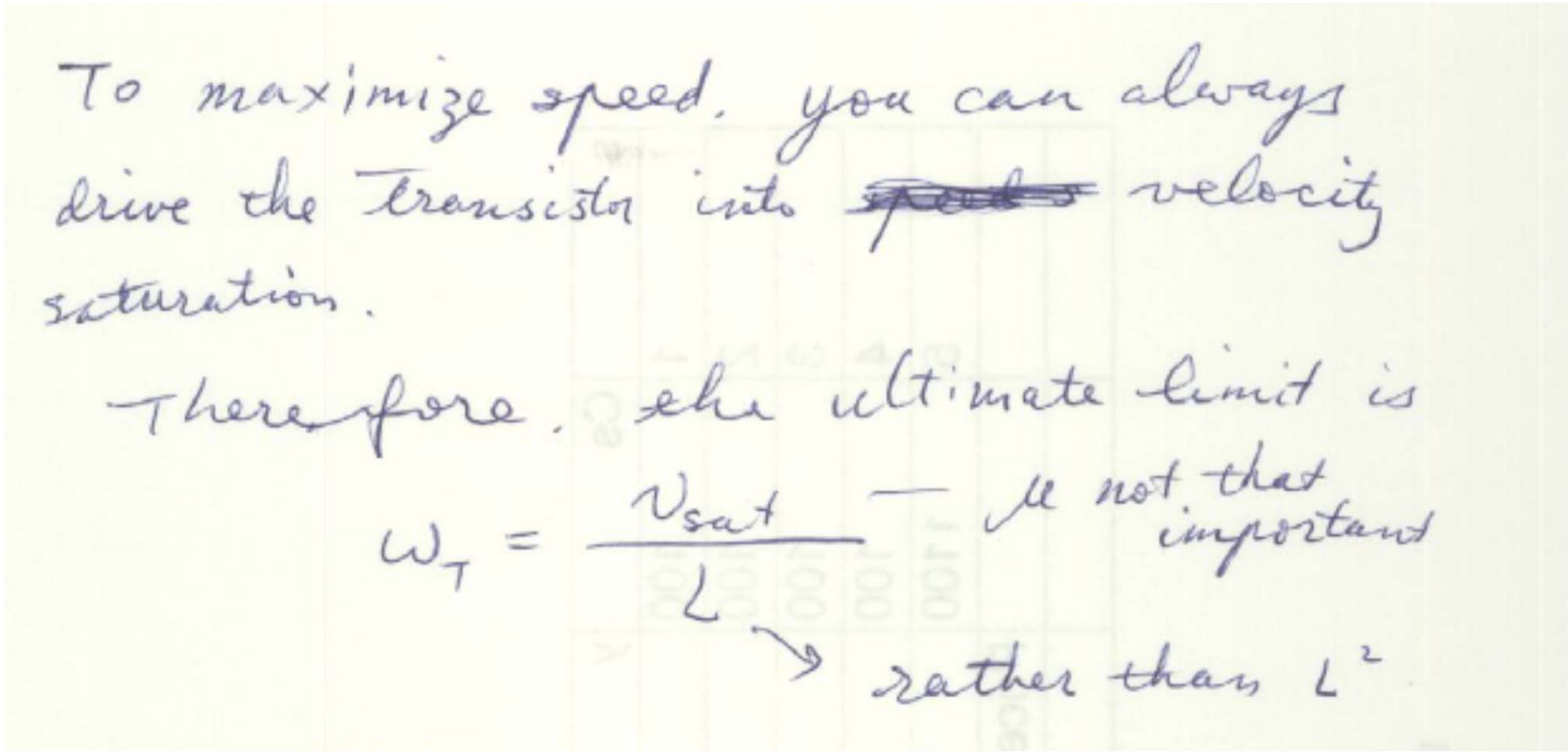
Considering velocity saturation,

$$I_D = Q \cdot \frac{v_{sat}}{L} = C_{ox} W (V_G - V_{tf}) v_{sat}$$

- $$g_m = \frac{dI_D}{dV_G} = C_{ox} W v_{sat}$$

$$\omega_T = \frac{g_m}{C_{gs}} = \frac{C_{ox} W v_{sat}}{C_{ox} W L} = \frac{v_{sat}}{L}$$

Whatever prefactor you have here will cancel



In addition, high mobility does not guarantee high v_{sat}.
High optical phonon energy is an indicator of high v_{sat}.

Further comments

- f_T is just a figure of merit.
  The transistor can't operate @ f_T.
- $f_T \propto \frac{1}{L^2}$ or $f_T \propto \frac{1}{L}$.
  shorter channel $\Rightarrow$ faster.
- $f_T \propto \mu$ or $f_T \propto v_{sat}$
  faster carriers $\Rightarrow$ faster transistor